J. Fluid Mech. (1995), vol. 301, pp. 19–35

Reconsideration of oblique shock wave reflections in steady flows. Part 1. Experimental investigation

By A. CHPOUN[1], D. PASSEREL[1], H. LI[2] AND G. BEN-DOR[2]

[1] Laboratoire d'Aerothermique du CNRS, Meudon, France

[2] Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer Sheva, Israel

(Received 10 May 1994 and in revised form 19 May 1995)

The reflection of shock waves over straight reflecting surfaces in steady flows was investigated experimentally using the supersonic wind tunnel of Laboratoire d'Aerothermique du CNRS, Meudon, France. The results for a flow Mach number $M_0 = 4.96$ contradict the state of the art regarding the regular$\leftrightarrow$Mach reflection transition in steady flows. Not only was a hysteresis found to exist in this transition, but, unlike previous reports, regular reflection configurations were found to be stable in the dual-solution domain in which theoretically both regular and Mach reflection are possible.

1. Introduction

As discussed by Ben-Dor (1991), two shock-wave-reflection configurations are possible in steady flows, namely regular reflection (RR) and Mach reflection (MR). Schematic illustrations of the wave configurations of a regular and a Mach reflection, together with the definitions of some flow parameters, are shown in figures 1(*a*) and 1(*b*), respectively.

The regular reflection (RR) consists of two shock waves, namely the incident shock wave, i, and the reflected shock wave, r. They meet at the reflection point, R, which is located on the reflecting surface. The flow states are (0) ahead of i, (1) behind it and (2) behind r. The angle of incidence, ϕ_1, of a regular reflection is sufficiently small so that the streamline deflection, θ_1, caused by the incident shock wave, i, can be cancelled by the opposite streamline deflection, θ_2, caused by the reflected shock wave, r. Therefore, the boundary condition of a regular reflection is

$$\theta_1 - \theta_2 = 0. \tag{1}$$

The Mach reflection (MR) consists of three shock waves, namely the incident shock wave, i, the reflected shock wave, r, and the Mach stem, m, and also one slipstream, s. They all meet at a single point known as the triple point, T. The Mach stem, m, is usually a curved shock wave which is perpendicular to the surface of the reflecting wedge at the reflection point R. The flow states are (0) ahead of i and m, (1) behind i, (2) behind r and (3) behind m. Unlike the case of a regular reflection where the net deflection of the streamlines is zero, in the case of a Mach reflection the net deflection of the streamlines is non-zero, in general, and the streamlines behind the triple point are directed towards the reflecting wall. Since the streamlines on both sides of the slipstream must be parallel to each other, the boundary condition of a Mach reflection is

$$\theta_1 - \theta_2 = \theta_3. \tag{2}$$

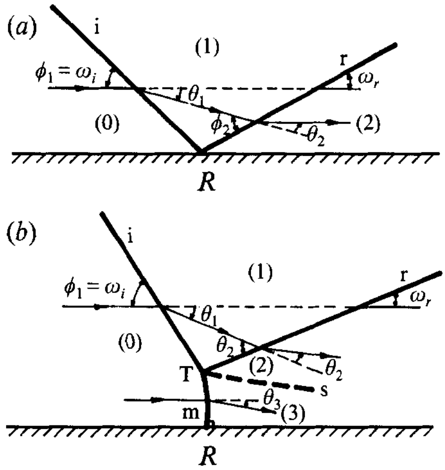

FIGURE 1. Schematic illustration of the wave configurations of (*a*) regular reflection and (*b*) Mach reflection. i, incident shock wave; r, reflected shock wave; m, Mach stem; s, slipstream; R, reflection point; T, triple point; ϕ_1 and ϕ_2, angles of incidence of i and r, respectively; θ_1, θ_2 and θ_3, flow deflections while passing through i, r and m, respectively; ω_i and ω_r, wave angles of i and r, respectively. Note that for both configurations $\omega_i = \phi_1$ and $\omega_r = \phi_2 - \theta_1$.

It should be noted here that equations (1) and (2) are based on local considerations in the vicinities of the reflection point of an RR and the triple point of an MR, respectively. In order for these conditions to be global, the discontinuities, i.e. shock waves and slipstreams, must be straight so that the flow regions bounded by them are uniform.

Graphical solutions in the pressure–deflection plane (i.e. the (P, θ)-plane) have been traditionally used to illustrate and better understand the shock wave reflection phenomenon, in general, and possible RR$\leftrightarrow$MR transition criteria, in particular. Examples of five different I–R polar combinations for increasing values of ϕ_1 are shown in figures 2(*a–e*). The loci of all the pressures achievable from the free stream of state (0) via an oblique shock wave deflecting the flow through an angle θ are given by the I-polar. Thus, state (1) in figures 1(*a*) or 1(*b*) maps into point (1) in figures 2(*a–e*). The loci of all the pressures achievable from the free stream of state (1) via an oblique shock wave deflecting it by an angle θ are given by the R-polar.

The boundary condition of a regular reflection, given by equation (1), implies that the solution of a regular reflection in the (P, θ)-plane is at the point where the R-polar intersects the P-axis. Two such points are obtained by the I–R polar combination shown in figure 2(*a*). It is an experimental fact that the one resulting in the higher pressure (marked by an open circle in figure 2*a*) is unstable. Li (1995) showed recently that the higher-pressure solution violates the principle of minimum entropy production. This fact might be used to conclude that the strong solution is aphysical. Consequently, state (2) of figure 1(*a*) maps into point (2) in figure 2(*a*).

The boundary condition of a Mach reflection, given by equation (2), implies that the solution of a Mach reflection is at the point where the I and R polars intersect. Such a point is indicated by the I–R polar combination shown in figure 2(*e*). States (2) and (3) of the Mach reflection, shown in figure 1(*b*), map into that point. Note that state (2) is on the R-polar and state (3) is on the I-polar.

Three intermediate I–R polar combinations are shown in figures 2(*b–d*). If one starts with initial conditions, i.e. M_0 and ϕ_1 (where M_0 is the flow Mach number and

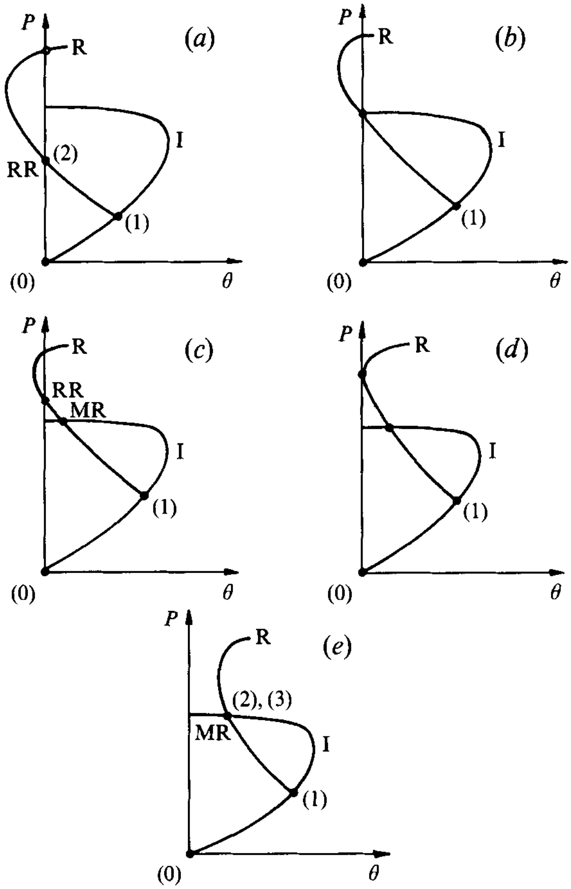

FIGURE 2. Various I–R polar combinations illustrating (*a*) a regular reflection, (*b*) the von Neumann criterion (also known as the mechanical equilibrium criterion), (*c*) regular and Mach reflection for the same initial conditions, (*d*) the detachment criterion, and (*e*) a Mach reflection.

ϕ_1 is the angle of incidence), appropriate to the regular reflection whose solution is presented by the I–R polar combination shown in figure 2(*a*), and then increases the angle of incidence ϕ_1 while keeping the uniform flow Mach number, M_0, constant, the I–R polar combination shown in figure 2(*b*) is eventually reached. Since for this combination the R-polar intersects both the *P*-axis and the I-polar, both RR and MR are theoretically possible at this intersection point. Furthermore, this I–R polar combination represents a possible condition for the RR$\leftrightarrow$MR transition. This possible transition, which was first suggested by von Neumann in the early 1940s (see von Neumann 1963) was re-introduced by Henderson & Lozzi (1975) who called it the 'mechanical equilibrium' criterion. It is known nowadays as the *von Neumann criterion* and can be formulated by combining equations (1) and (2) to read

$$\theta_1 - \theta_2 = \theta_3 = 0. \qquad (3)$$

As can be seen in figure 2(*b*) the Mach stem in the vicinity of the triple point is normal to the oncoming flow. The incident shock wave angle appropriate to the I–R polar combination shown in figure 2(*b*) will be denoted as ω_i^N. Note that for $\omega_i < \omega_i^N$ a Mach reflection is impossible, hence ω_i^N is the smallest incident shock wave angle for which a Mach reflection is possible for a given flow Mach number M_0.

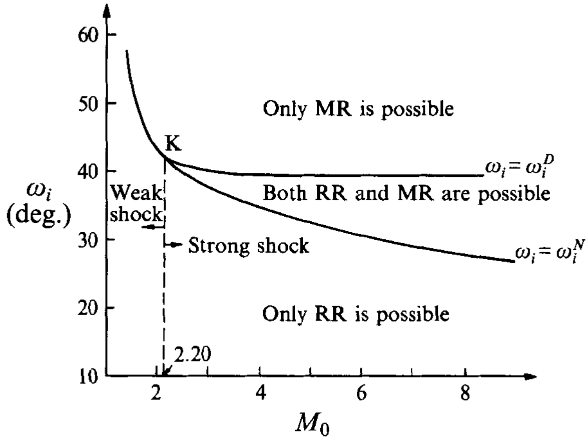

FIGURE 3. Domains of possible reflection configurations in the (M_0, ω_i)-plane at $\gamma = 1.4$. $\omega_i = \omega_i^D$ and $\omega_i = \omega_i^N$ are the transition lines corresponding to the detachment and von Neumann criteria, respectively.

A further increase in the incident shock wave angle eventually results in the I–R polar combination shown in figure 2(*d*) which, in fact, corresponds to the largest value of ω_i for which a regular reflection is obtainable for a given flow Mach number M_0. Note that an increase of ω_i beyond the value appropriate to that of figure 2(*d*) results in a situation similar to that shown in figure 2(*e*), in which the R-polar does not intersect the *P*-axis and hence a regular reflection is impossible. Consequently, the I–R polar combination of figure 2(*d*) represents another possible condition for the RR$\leftrightarrow$MR transition. The possible transition, which was also suggested by von Neumann (1963), is known as the *detachment criterion* because it corresponds to the case in which the streamline deflection through the reflected shock wave is maximal. Its mathematical formulation is

$$\theta_1 - \theta_2^D = 0, \tag{4}$$

where θ_2^D is the detachment deflection angle. The incident shock wave angle appropriate to the I–R polar combination shown in figure 2(*d*) will be denoted as ω_i^D. Note that for $\omega_i > \omega_i^D$ a regular reflection is impossible, hence ω_i^D is the largest incident shock wave angle for which a regular reflection is possible for a given flow Mach number M_0.

Based on the foregoing discussion, for all the incident shock wave angles in the range

$$\omega_i^N \leqslant \omega_i \leqslant \omega_i^D$$

both regular and Mach reflections are possible. A typical I–R polar combination appropriate to this dual-solution domain is shown in figure 2(*c*). The point where the R-polar intersects the *P*-axis indicates a possible RR solution, while the point where it intersects the I-polar indicates a possible MR solution.

It should be noted here that since both RR and MR are possible for the dual-solution domain given by $\omega_i^N \leqslant \omega_i \leqslant \omega_i^D$, the RR$\leftrightarrow$MR transition could occur at any value of ω_i inside that range. Consequently, the von Neumann criterion, $\omega_i = \omega_i^N$, is the lowest possible value of ω_i for transition, while the detachment criterion, i.e. $\omega_i = \omega_i^D$, is the largest possible value of ω_i for transition.

It should also be noted here that while transition at angles of incidence in the range $\omega_i^N < \omega_i \leqslant \omega_i^D$ involves a sudden pressure change behind the reflected shock wave (i.e.

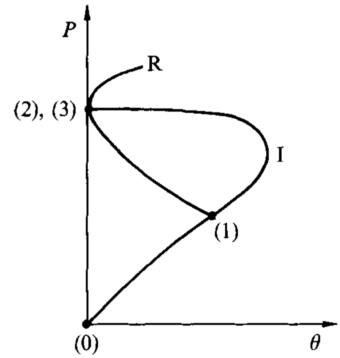

FIGURE 4. The I–R polar combination corresponding to point K is figure 3.

positive for the MR→RR transition and negative for the RR→MR transition), a transition at $\omega_i = \omega_i^N$ is continuous as far as the pressure behind the reflected shock wave is concerned.

The dual-solution domain for which both regular and Mach reflections are possible in the (M_0, ω_i)-plane is shown in figure 3 for $\gamma = 1.4$, where γ is the ratio of the specific heat capacities. The detachment $(\omega_i = \omega_i^D)$ and the von Neumann $(\omega_i = \omega_i^N)$ criteria divide the (M_0, ω_i)-plane into three domains: a domain in which only RR is theoretically possible $(\omega_i < \omega_i^N)$, a domain in which only MR is theoretically possible $(\omega_i > \omega_i^D)$, and a domain in which both RR and MR are theoretically possible $(\omega_i^N \leqslant \omega_i \leqslant \omega_i^D)$.

While the detachment criterion exists for all values of $M_0 > 1$, the von Neumann criterion does not exist in the range $M_0 \leqslant 2.20$. The point $M_0 = 2.20$ is the point, marked by K in figure 3, where the two transition lines arising from these two transition criteria meet. Traditionally, it has been used to distinguish between weak and strong incident shock waves. The I–R polar combination appropriate to the conditions of point K is shown in figure 4.

The RR↔MR *transition – state of the art*

Hornung & Robinson's (1982) conclusions have been accepted in the scientific community as the state of the art regarding the RR↔MR transition in steady flows.

Based on their own experiments, as well as those of Henderson & Lozzi (1975, 1979) and Hornung & Kychakoff (1977), they concluded that '...in steady flow, the transition from regular to Mach reflection of strong shock waves [i.e. $M_0 > 2.20$] occurs at the von Neumann condition [i.e. $\omega_i = \omega_i^N$] and not at the detachment condition [i.e. $\omega_i = \omega_i^D$]...' as has been mentioned in well-known textbooks such as Liepmann & Roshko (1957), Landau & Lifshitz (1987) and Anderson (1982).

The above conclusion reached by Hornung & Robinson (1982) was based on experiments which they conducted in the range $2.8 \leqslant M_0 \leqslant 5$ where, as can be seen from figure 3, the difference between the transition lines appropriate to $\omega_i = \omega_i^N$ and $\omega_i = \omega_i^D$ is sufficiently large for clearly distinguishing between them.

Based on their lengthscale concept Hornung, Oertel & Sandeman (1979) hypothesized that a hysteresis should exist in the RR↔MR transition. Consider figure 3 and assume that one starts with an MR at $\omega_i > \omega_i^D$ and then slowly decreases ω_i while M_0 is kept constant. As a result, the Mach stem height decreases until it vanishes at $\omega_i = \omega_i^N$ where a smooth transition from MR to RR takes place. If the decrease in ω_i continues, then RR is maintained. If now ω_i is slowly increased, then, since the flow

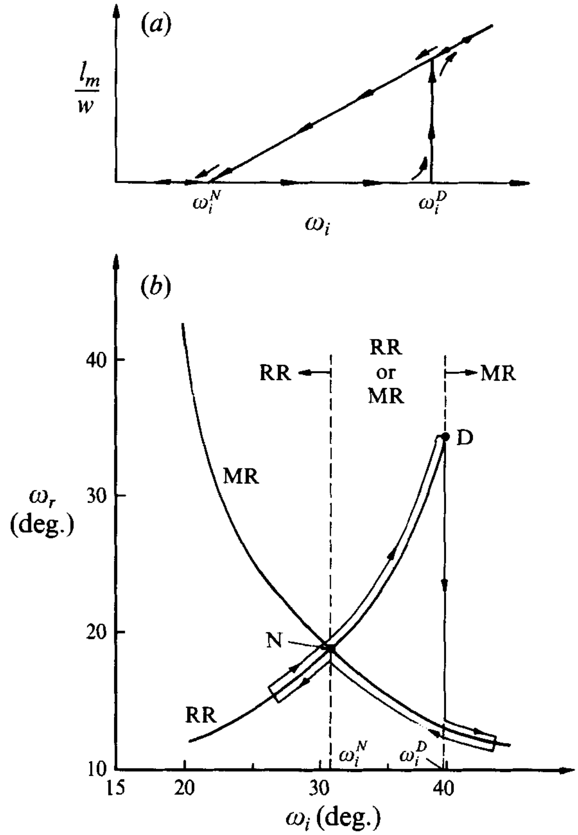

FIGURE 5. Schematic illustrations on the possible hysteresis in the RR$\leftrightarrow$MR transition: (*a*) in the (ω_i, l_m)-plane, and (*b*) in the (ω_i, ω_r)-plane ($M_0 = 4.96$ and $\gamma = 1.4$).

field is free of significant disturbances, the RR should be maintained until $\omega_i = \omega_i^D$ where a sudden transition from RR to MR should occur. The above-suggested hysteresis loop is shown in the (ω_i, l_m)- and the (ω_i, ω_r)-planes in figures 5(*a*) and 5(*b*), respectively; ω_i and ω_r are the angles which the incident and reflected shock waves make with the horizontal axis as shown in figures 1(*a*) and 1(*b*), and l_m is the Mach stem height. Hornung & Robinson's (1982) experimental attempts to confirm their hypothesis regarding the hysteresis in the RR$\leftrightarrow$MR transition failed and they concluded that 'the hysteresis effect predicted by Hornung *et al.* (1979)...could not be confirmed'. For this reason they refer to the RR in the dual solution domain, $\omega_i^N < \omega_i < \omega_i^D$, as an unstable regular reflection.

The following summarizes the state of the art regarding the RR$\leftrightarrow$MR transition of planar shock waves in steady flows over straight wedges:

(i) the RR$\rightarrow$MR and the MR$\rightarrow$RR transitions occur at the von Neumann condition, i.e. at $\omega_i = \omega_i^N$;

(ii) the RR in the dual-solution domain, i.e. $\omega_i^N < \omega_i < \omega_i^D$, is unstable;

(iii) a hysteresis phenomenon does not exist in the RR$\leftrightarrow$MR transition.

It is very important to note here that the above conclusions were based on two sets of experiments, one of which was conducted by Henderson's research group and the other by Hornung's, and there were no experimental reasons to doubt these conclusions. However, in a recent analytical study, Li (1995) showed, by applying

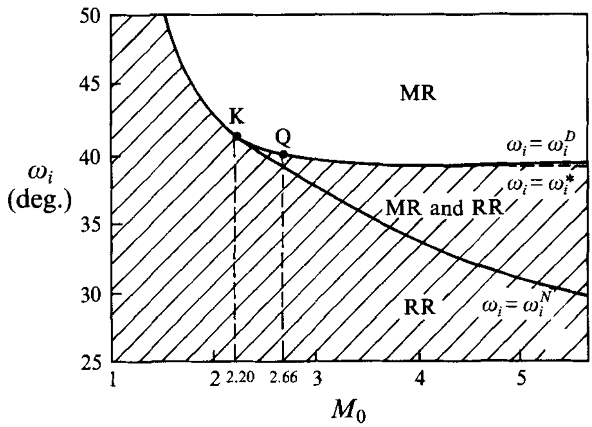

FIGURE 6. Schematic illustration of the domain of stable RR configurations in the (M_0, ω_i)-plane at $\gamma = 1.4$. The RR is stable for $\omega_i < \omega_i^*$.

the principle of minimum entropy production that, contrary to Hornung & Robinson's (1982) conclusion, the RR in most of the dual-solution domain, $\omega_i^N \leqslant \omega_i \leqslant \omega_i^D$, is stable.

Figure 6 is based on figure 3. In addition to the $\omega_i = \omega_i^N$ and $\omega_i = \omega_i^D$ transition lines it also includes the curve $\omega_i = \omega_i^*$. The domain $\omega_i \leqslant \omega_i^*$ satisfies the principle of minimum entropy production. This fact might be used as an indication that RR wave configurations are physical in that domain. Note that the $\omega_i = \omega_i^*$ line is located very close to the $\omega_i = \omega_i^D$ line. Consequently, of the entire $\omega_i^N \leqslant \omega_i \leqslant \omega_i^D$ domain for which RR is theoretically possible, only in a very narrow domain, $\omega_i^* < \omega_i \leqslant \omega_i^D$, is RR aphysical.

For this reason it was decided to conduct a detailed experimental investigation of the RR$\leftrightarrow$MR transition of planar shock waves over straight reflecting surfaces in steady flows in an attempt to establish a stable regular reflection inside the dual-solution domain, $\omega_i^N \leqslant \omega_i \leqslant \omega_i^D$. As will be shown subsequently, not only did we succeed in achieving this goal, but our experimental results contradict the above-listed three conclusions, which constitute the state of the art regarding the RR$\leftrightarrow$MR transition in steady flows. Consequently, we propose a different transition criterion which, to the best of our knowledge, agrees with all the relevant available experiments.

2. Present experimental study

2.1. *The experimental facility*

The experiments were conducted on the SH2 supersonic wind tunnel of Laboratoire d'Aerothermique du CNRS, Meudon, France. The SH2 wind tunnel is an open jet continuous facility. The wind tunnel run time is virtually infinite. The diameters of the exit and the jet sections are 127 mm and 120 mm, respectively. The ratio between the area of the exit and the throat sections is 25. Consequently, the flow Mach number in the test section based on inviscid theory was designed to be exactly $M_0 = 5$. However, owing to viscous effects the actual flow Mach number was $M_0 = 4.96$. This value was maintained within a variation of less than 1 % along a jet length of about 200 mm. The stagnation pressure and temperature were $P_t = 8.5$ bar and $T_t = 453$ K, respectively. These conditions give a Reynolds number per unit length of 1.3×10^7 m^{-1}. The mass

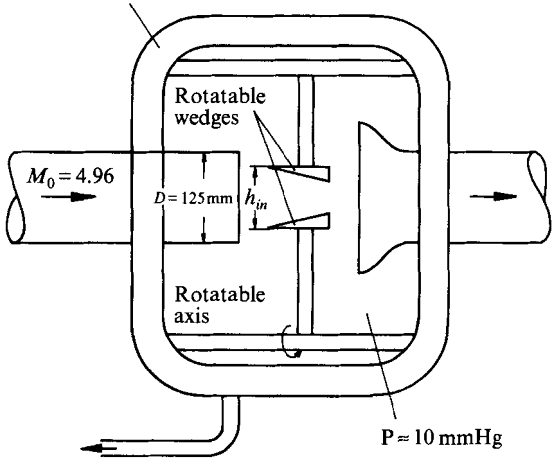

FIGURE 7. Schematic illustration of the test section and the experimental set-up used in the present experimental study.

flow rate was 0.76 kg s^{-1}. Two stage compressors, MPR-RC300, supplied the air at a pressure of 10 bar. The flow was heated up to a stagnation temperature of $T_t = 453$ K to avoid liquefaction during the expansion in the nozzle.

A group of MPR-P600 and MPR-P1000 pumps maintained a pressure of 10 mmHg (below the static pressure of the supersonic jet) inside the chamber enclosing the supersonic jet. When models were not used the pressure inside the chamber was 4 mmHg.

A schematic illustration of the experimental model inside the chamber is shown in figure 7. In order to avoid possible boundary layer influence on the RR$\leftrightarrow$MR transition double-wedge models in a symmetrical configuration were used. Note that, as argued by Hornung & Robinson (1982), this configuration could not avoid the viscous growth of the shear layers on both sides of the slipstream; these were considered by Ben-Dor (1987). However, since this shear layer is associated with an MR it is the authors' belief that if it has any effect on the RR$\rightarrow$MR transition the effect is minimal. It could, however, affect the opposite MR$\rightarrow$RR transition. The double-wedge model did not span the cross-section of the supersonic jet. This set-up was chosen in order to avoid the interaction of the shock waves with the sidewall boundary layers. The model formed a two-dimensional converging nozzle which was terminated by a throat formed by the trailing edges of the reflecting wedges. Because the model did not span the tunnel test section, air could be ejected from it sideways, particularly near the throat. It is believed that this mechanism stabilized the wave system which developed. Note that a similar set-up was used in Henderson & Lozzi's (1975, 1979) investigations.

Both the upper and the lower reflecting wedges were connected to an electric motor which could place the plates at any fixed angle or could continuously rotate them with a rate of rotation of about 0.57° s^{-1}. The two wedges were mounted on a rotational mechanism which kept the inlet cross-section, h_{in}, constant during the rotation.

In addition, the wedges could be tilted sideways by 90° and thereby completely removed from the jet section. The reason for enabling this degree of freedom is given subsequently.

Colour schlieren photography was employed to record the reflection phenomenon both with still (single shot) and continuous (video) cameras.

2.2. *The experimental procedure*

When the wind tunnel was started the wedges were placed outside the test section (i.e. the jet section). After a constant flow was established and the nominal conditions were obtained the wedges were brought into the flow one after the other. Then the hysteresis experiments were carried out in the following two manners.

Discrete variation of the wedge angle

The wedge angles were set to a succession of prescribed values ranging from small angles which resulted in regular reflections, to larger angles which resulted in Mach reflections, and then from large to small values. For each position a video picture was digitized, stored in a computer, then printed and evaluated. The time interval between two picture acquisitions was typically three minutes which was much larger than the flow establishment time. Consequently, there was no doubt that at each position a true steady flow was established over the wedge.

Continuous variation of the wedge angle

The wedge angles were changed continuously at a very slow rate of about $0.57° \mathrm{s}^{-1}$ and a video movie was recorded.

2.3. *Experimental results*

The RR$\leftrightarrow$MR *transition*

The first set of experiments was aimed at determining the conditions under which the RR$\leftrightarrow$MR transition occurred.

Initially an MR at $\omega_i \approx 40°$ (recall that in all the experiments $M_0 = 4.96$) was established. Then the angles of attack of the two reflecting wedges were simultaneously and continuously decreased at a rate of about $0.57° \mathrm{s}^{-1}$. As a result, the wave angles, ω_i, of the incident shock waves decreased. This resulted in a continuous decrease in the height of the Mach stem until a situation in which the Mach stem completely vanished was reached. The video movie clearly indicated that the MR$\rightarrow$RR transition was completely smooth. The point where this occurred was defined as the experimental MR$\rightarrow$RR transition point.

The angles of attack of the reflecting wedges were then further decreased until the wave angles of the incident shock waves reached values of $\omega_i \leqslant 30°$ which, as can be seen in figure 3, are well inside the domain in which, theoretically, only RR was possible. Then the direction of the rotation of the wedges was reversed and their angles of attack were simultaneously and continuously increased.

While increasing the angles of attack it was observed that the RR wave configuration was not terminated at the earlier determined MR$\rightarrow$RR experimental transition point. Instead the RR wave configuration was maintained for some time until it suddenly changed to an MR. The point where this occurred was defined as the experimental RR$\rightarrow$MR transition point. The height of the Mach stem of the suddenly formed MR was definitely non-zero.

The above-described experimental results are shown in figure 8 in the (ω_i, ω_r)-plane for $h_{in} = 9$ cm and wedge surface length $w = 6$ cm. MR wave configurations are marked with triangles and RR wave configurations are marked with circles. The von Neumann and the detachment transition angles for $M_0 = 4.96$ are $\omega_i^N = 30.88°$ and $\omega_i^D = 39.33°$, respectively.

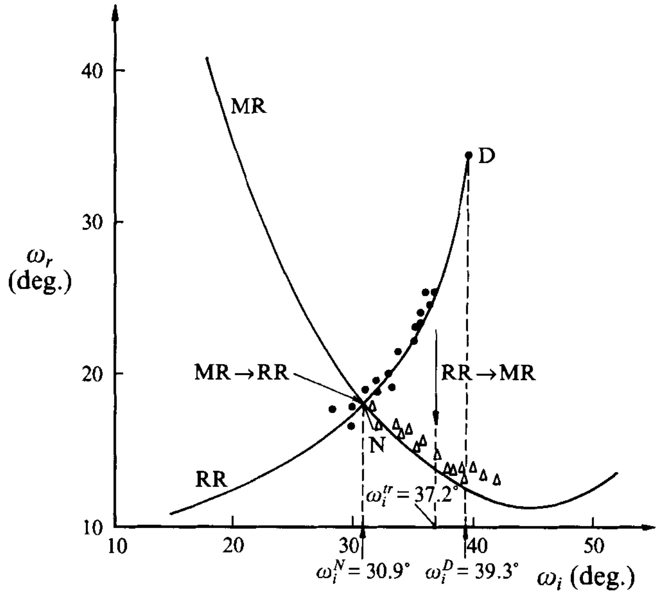

FIGURE 8. The present experimental results in the (ω_i, ω_r)-plane: Δ, MR configurations; $\bigcirc$, RR configurations. The hysteresis phenomenon in the RR↔MR transition is clearly evident.

The hysteresis phenomenon in the RR↔MR transition which was shown schematically in figure 5(*b*) is clearly evident in figure 8. While the MR→RR transition took place at the von Neumann angle, i.e. $\omega_i^{tr}(\mathrm{MR}\rightarrow\mathrm{RR}) = \omega_i^N = 30.9°$, the reverse transition, the RR→MR transition, occurred at about $\omega_i^{tr}(\mathrm{RR}\rightarrow\mathrm{MR}) = 37.2°$. Thus, it is clear that the hysteresis phenomenon which was hypothesized by Hornung *et al.* (1979) and up to now has not been confirmed experimentally does exist in the RR↔MR transition in steady flows. It should be noted here that the hysteresis was also evident in the experiments in which the wedge angles were changed in discrete steps.

Note also that the experimental results shown in figure 8 are consistent with the general transition criteria which were put forward recently by Li (1995).

The dependence of the transition on geometrical parameters

In order to investigate the effect of the geometrical set-up (i.e. lengthscale effect) on the transition process, experiments similar to the one mentioned above were repeated for three different reflecting wedge surface lengths, w (50, 60 and 70 mm). Each reflecting wedge was tested with three different combinations of inlet cross-sections, h_{in} (about 70, 85 and 100 mm). In seven out of the above nine possible combinations of w and h_{in} both the RR→MR and the MR→RR processes were recorded by setting the reflecting wedge angles to a succession of prescribed values. For each angle the video picture was digitized, stored in the computer, printed and evaluated. The experiments were also repeated while taking a continuous movie record. The results of these experiments, which are shown in figure 9, again verify the above-mentioned hysteresis phenomenon in the transition process. All the open symbols in which the MR→RR transition was investigated show transition in the vicinity of the von Neumann transition angle ω_i^N. On the other hand, all the solid symbols in which the RR→MR transition was investigated indicate that the transition occurs between the von Neumann, ω_i^N, and the detachment, ω_i^D, transition angles, in the vicinity of $\omega_i = 37°$.

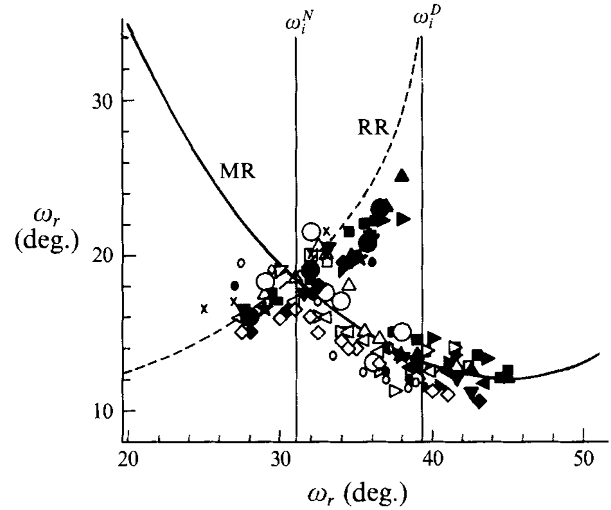

FIGURE 9. The present experimental results for the nine combinations of w and h_{in} in the (ω_i, ω_r)-plane. Open symbols MR $\rightarrow$ RR process. Solid and × symbols RR $\rightarrow$ MR process. □, ■, $w = 50$ mm, $h_{in} = 99.5$ mm; △, ▲, $w = 50$ mm, 85.2 mm; ▼, $w = 50$ mm, $h_{in} = 69.6$ mm; ▷, ▶, $w = 60$ mm, $h_{in} = 99.5$ mm; ◁, ◀, $w = 60$ mm, $h_{in} = 85$ mm; ○, ●, $w = 60$ mm, $h_{in} = 69.8$ mm; ◇, ◆, $w = 70$ mm, $h_{in} = 99.5$ mm; ○, •, $w = 70$ mm, $h_{in} = 85$ mm; ×, $w = 70$ mm, $h_{in} = 69$ mm.

In addition, it is clearly indicated in figure 9 that the RR $\rightarrow$ MR transition depends slightly on both the inlet cross-section, h_{in}, and the reflecting wedge surface length, w.

The stability of regular and Mach reflections

The stability of both regular and Mach reflection wave configurations inside the dual solution was examined in the following way. The experiment in which the wedge angle was changed in discrete steps was repeated. The change in the wedge angle between two successive positions was about 1°. At each position the holding arm of the lower wedge (of the two symmetrical wedges) was tilted sideways by 90° and completely removed from the flow field. This resulted in a situation in which only an oblique straight shock wave emanating from the leading edge of the upper wedge was left in the flow field. At this stage the lower wedge was brought back to its original position and the flow was allowed to reach its steady conditions. It was found that there is a critical incident shock wave angle value, say ω_i^{cr}, below which the stable reflection was found to be RR and above which the stable reflection was found to be MR. For the wedge with $h_{in} = 9$ cm and $w = 6$ cm, for which results are shown in figure 8, it was found that $\omega_i^{cr} \approx 35.5°$. Consequently, from a stability point of view the dual-solution domain can be divided into two sub-domains: RR wave configurations are stable in the sub-domain $\omega_i^N \leqslant \omega_i < \omega_i^{cr}$; MR wave configurations are stable in the sub-domain $\omega_i^{cr} < \omega_i \leqslant \omega_i^{tr}$ (RR $\rightarrow$ MR). Owing to the fact that ω_i^{tr}(RR $\rightarrow$ MR) was found to depend on geometrical parameters, it is hypothesized here that ω_i^{cr} also depends on the geometrical parameters associated with the experiment.

The fact that RR wave configurations were established in spite of the very large disturbance associated with this experimental procedure contradicts the conclusion of Hornung *et al.* (1979) that inside the dual-solution domain large disturbances should be sufficient to cause the reflection to be an MR rather than an RR.

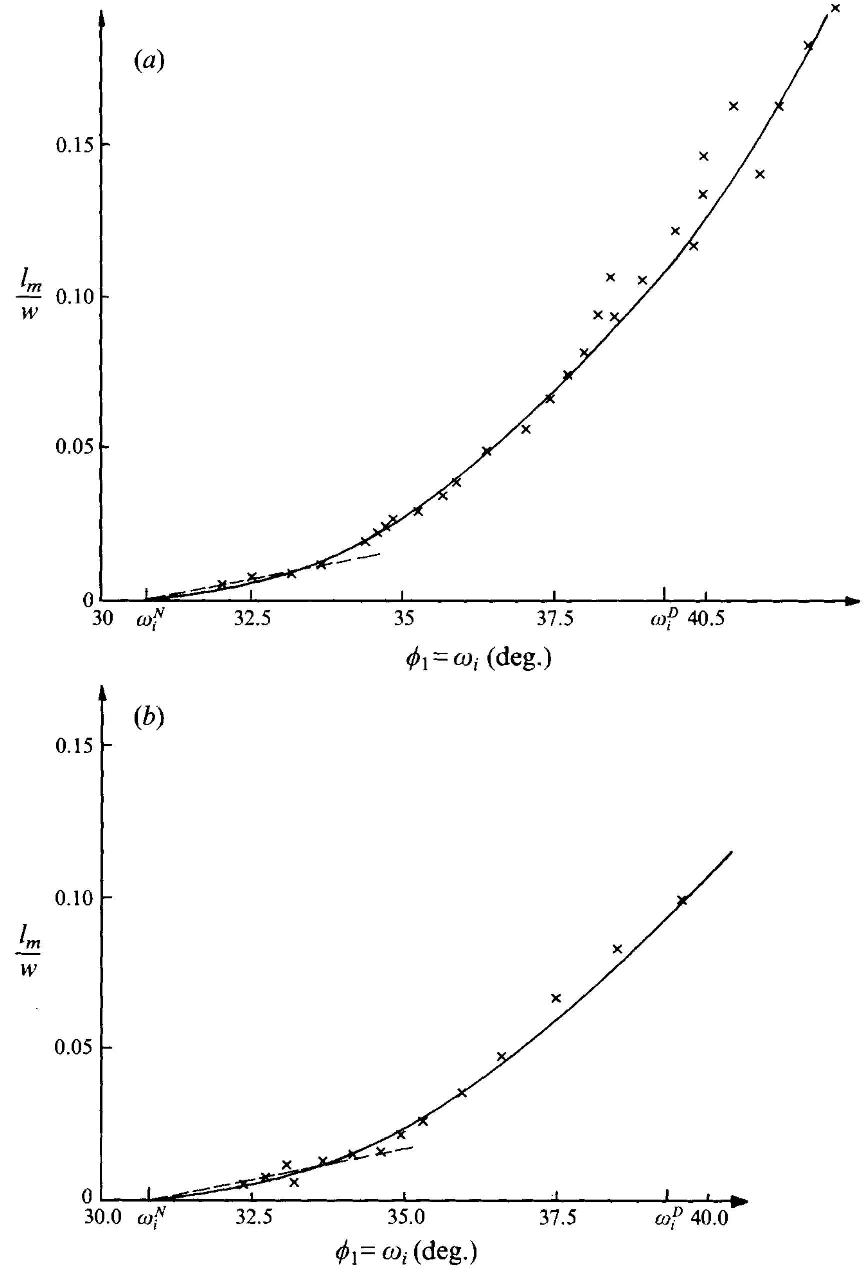

FIGURE 10. Dependence of the height of the Mach stem of a Mach reflection on the wave angle of the incident shock wave, for (*a*) $h_{in} = 9$ cm, and (*b*) $h_{in} = 10$ cm. For both cases $w = 6$ cm.

Some further notes regarding the hysteresis

Consider figures 10(*a*) and 10(*b*) where the dependence of the non-dimensionalized Mach stem height l_m/w (l_m is the height of the Mach stem and w is the length of the reflecting wedge) on the angle of incidence of the oncoming flow ω_i is shown for two different inlet cross-sectional areas h_{in}. The solid lines are least-square fits to second-order polynomials. As can be seen the value of ω_i at the point where the fitted curves

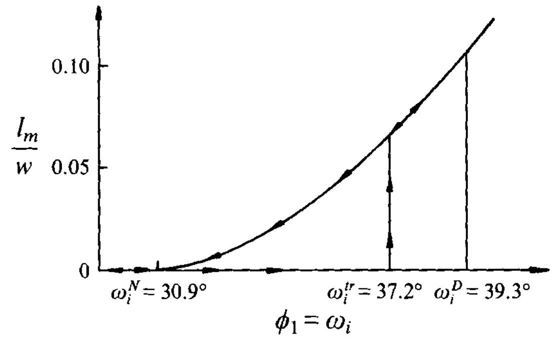

FIGURE 11. The actual hysteresis loop in the (ω_i, l_m)-plane, for $h_{in} = 9$ cm and $w = 6$ cm indicated by the experiments shown in figure 10(*a*).

approached $l_m \rightarrow 0$ is exactly equal to the von Neumann angle ω_i^N. Hence, based on figures 10(*a*) and 10(*b*) there is little doubt that the MR → RR transition occurs at the von Neumann angle, i.e. $\omega_i^{tr}(\text{MR} \rightarrow \text{RR}) = \omega_i^N$. Furthermore, the transition angle seems to be independent of the inlet cross-sectional area h_{in}. In addition, the possible effect of the viscous growth on both sides of the slipstream on the MR → RR transition seems to be negligible.

It should be noted here that unlike Hornung & Robinson's (1982) experimental results where l_m/w was found to depend linearly on the incident shock wave angle ω_i, the present results reveal a nonlinear dependence. The reason for this difference lies simply in the fact that Hornung & Robinson's (1982) experiments were limited to a relatively narrow range of ω_i. For example, in their experiments with $M_0 = 5$ the investigated ω_i domain was $30.8° = \omega_i^N \leqslant \omega_i < 34°$. Our results in this narrow domain also resemble a linear dependence (see the dashed lines in figures 10*a* and 10*b*). However, when the entire range of $30.9° = \omega_i^N < \omega_i < \omega_i^D = 39.3°$ is considered, a clear nonlinear dependence of l_m on ω_i is evident. It should also be noted here that Azevedo & Liu (1993) in their ingenious attempt to analytically predict the Mach stem height of MR wave configurations in steady flows, also obtained a nonlinear dependence of l_m on ω_i. Their analytical curves are similar to those shown in figures 10(*a*) and 10(*b*). The actual hysteresis loop in the (l_m, ω_i)-plane is shown for the readers' convenience in figure 11. Note that unlike the loop shown in figure 5(*b*), here l_m does not decrease linearly with ω_i and the RR → MR transition takes place at $\omega_i \approx 37.2°$ rather than at $\omega_i = \omega_i^D = 39.3°$.

Schlieren photographs of the RR and MR wave configurations illustrating the above-mentioned hysteresis phenomenon are shown in figure 12. Figure 12(*a*) shows an MR at $\omega_i \approx 42° > \omega_i^D$. When ω_i was decreased the MR was maintained as shown in figure 12(*b*) where $\omega_i^N < \omega_i \approx 34.5° < \omega_i^D$. When ω_i decreased below ω_i^N an RR was obtained as shown in figure 12(*c*) for $\omega_i \approx 29.5° < \omega_i^N$. When the process was reversed and ω_i was increased beyond ω_i^N to a value of $\omega_i^N < \omega_i \approx 34.5° < \omega_i^D$, the reflection was RR as shown in figure 12(*d*). Note that the MR shown in figure 12(*b*) and the RR shown in figure 12(*d*) had practically the same initial conditions, i.e. $M_0 = 4.96$ and $\omega_i \approx 34.5°$. The fact that both of them were stable clearly verifies the fact that a hysteresis exists in the transition phenomenon. When ω_i further increased, the RR suddenly terminated and an MR was formed as shown in figure 12(*e*) where $\omega_i \approx 37.5° < \omega_i^D$.

The above-described experimental results also contradict the Henderson & Lozzi (1975) conclusion that the RR → MR transition should occur at the von Neumann

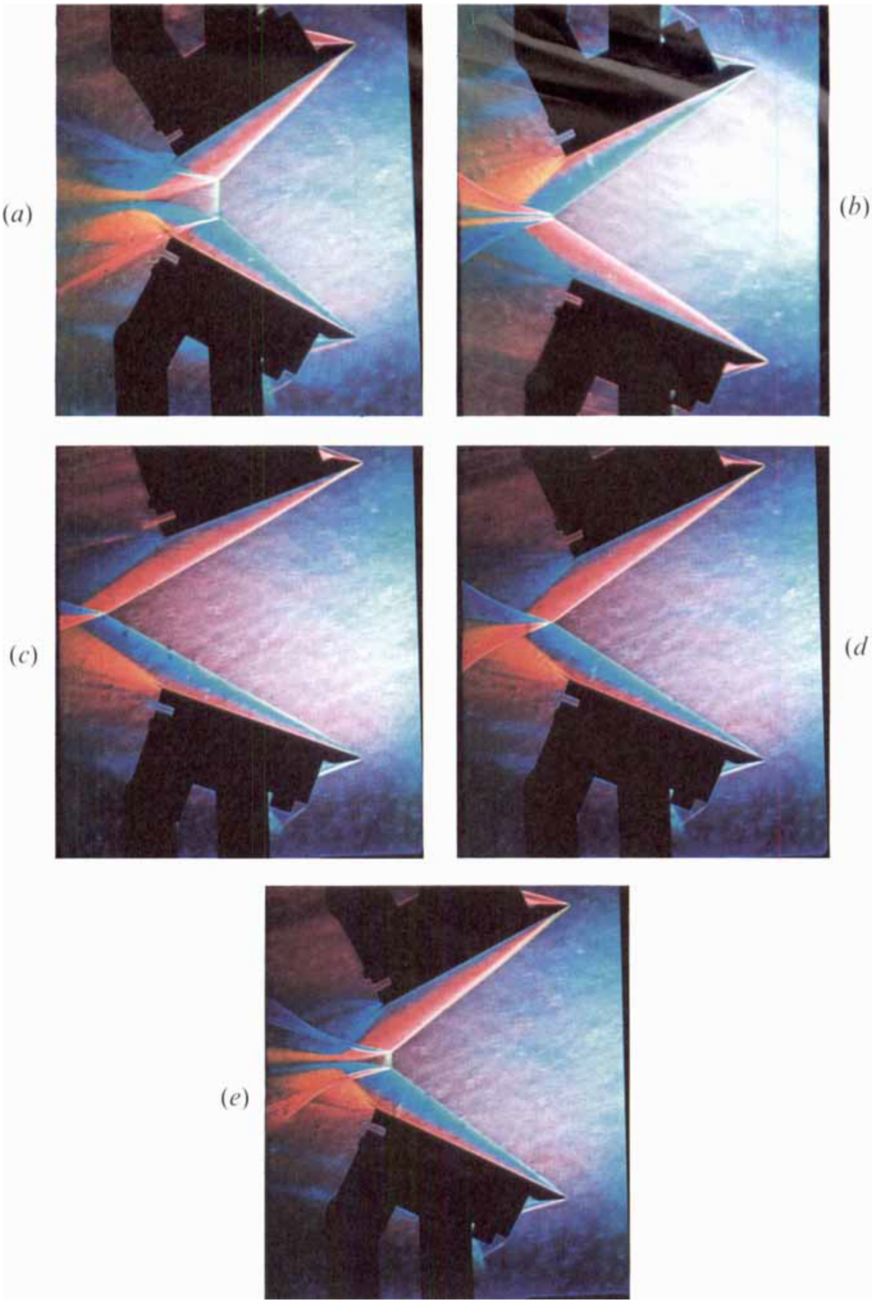

FIGURE 12. Schlieren photographs of (*a*) MR at $\omega_i \approx 42°$, (*b*) MR at $\omega_i \approx 34.5°$, (*c*) RR at $\omega_i \approx 29.5°$, (*d*) RR at $\omega_i \approx 34.5°$, and (*e*) MR at $\omega_i \approx 37.5°$.

condition, i.e. $\omega_i = \omega_i^N$, 'in such a way that mechanical equilibrium of the system is preserved through the process'. The sudden transition from RR to MR at $\omega_i > \omega_i^N$ occurred at a point where the mechanical equilibrium requirement was clearly not fulfilled.

3. Conclusions

An experimental investigation regarding the RR$\leftrightarrow$MR transition in steady flows revealed the following facts:

(i) A hysteresis exists in the RR$\leftrightarrow$MR transition. While the MR$\rightarrow$RR transition occurs at the von Neumann criterion, i.e. ω_i^{tr}(MR$\rightarrow$RR) $= \omega_i^N$, the reverse transition, i.e. the RR$\rightarrow$MR transition, can occur anywhere inside the dual-solution domain $\omega_i^N \leqslant \omega_i^{tr}$(RR$\rightarrow$MR) $\leqslant \omega_i^D$. The exact point at which the RR$\rightarrow$MR transition takes place depends probably on geometrical parameters associated with the experimental facility in which the experiments are carried out and the specific experimental set-up. It is possible that the experimental facilities and set-ups used by both Hornung and Henderson in their experimental investigations were such that the RR$\rightarrow$MR transition in their facilities occurred at or near the $\omega_i = \omega_i^N$ point. For this reason they failed to obtain stable RR wave configurations in the dual-solution domain and to observe the hysteresis phenomenon associated with the RR$\leftrightarrow$MR transition.

It should be mentioned here that by using curved reflecting wedges Henderson & Lozzi (1979) also obtained a hysteresis phenomenon in the RR$\leftrightarrow$MR transition. Their results are reproduced in figure 13. A comparison of their results which were obtained with curved reflecting wedges with our results which were obtained with straight reflecting wedges (see figure 8) indicates that, while their hysteresis loop takes place at $\omega_i < \omega_i^N$, ours occurs at $\omega_i > \omega_i^N$. Furthermore, their attempt to record a hysteresis phenomenon with straight wedges, similar to that recorded by us, failed. It should also be noted here that Ben-Dor, Takayama & Kawauchi (1980) recorded experimentally a similar hysteresis phenomenon when planar shock waves reflected over concave and convex cylindrical wedges. Their experiments revealed that:

$$\omega_i^{tr}(\mathrm{MR}\rightarrow\mathrm{RR}) = \omega_i^N \quad \text{and} \quad \omega_i^{tr}(\mathrm{RR}\rightarrow\mathrm{MR}) = \omega_i^D.$$

(ii) For the geometrical parameters of the wind tunnel and of the wedges and the experimental set-up used in the present study ω_i^{tr}(RR$\rightarrow$MR) $= 37.2° \pm 1.5°$ (recall that $M_0 = 4.96$). This violates Henderson & Lozzi's (1975) argument that the transition should occur at a point where mechanical equilibrium is preserved.

(iii) It was found that there is a critical value, ω_i^{cr}, below which RR wave configurations are stable and above which MR wave configurations are stable. These results contradict Hornung *et al.*'s (1979) explanation that if through some disturbance, for example, during the tunnel starting process, a Mach reflection is temporarily established in the $\omega_i^N \leqslant \omega_i \leqslant \omega_i^D$ domain, it will be stable. Consequently, either their 'lengthscale' concept is violated by the present experimental results or the large disturbances introduced by us were not sufficient to set up a temporary Mach reflection which is a condition for obtaining a stable MR in the dual-solution domain.

(iv) For the geometrical parameters of the wind tunnel and of the wedges and the experimental set-up used in the present study $\omega_i^{cr} \approx 35.5° \pm 1°$.

In summary, the experimental results which are presented in this paper contradict the state of the art regarding the RR$\leftrightarrow$MR transition in steady flows. However, they completely agree with the general analytical transition criteria which were suggested recently by Li (1995).

It should be noted here that based on their lengthscale concept Hornung *et al.* (1979) concluded that 'in steady or pseudo-steady flow, transition to Mach reflection occurs at that angle $\alpha_{tr} > \alpha_N$ [i.e. ω_i^{tr}(RR$\rightarrow$MR) $\geqslant \omega_i^N$] at which conditions change in such a way as to open an information path enabling the communication of a length scale to the reflection point'. Unfortunately, the geometrical parameters of their exper-

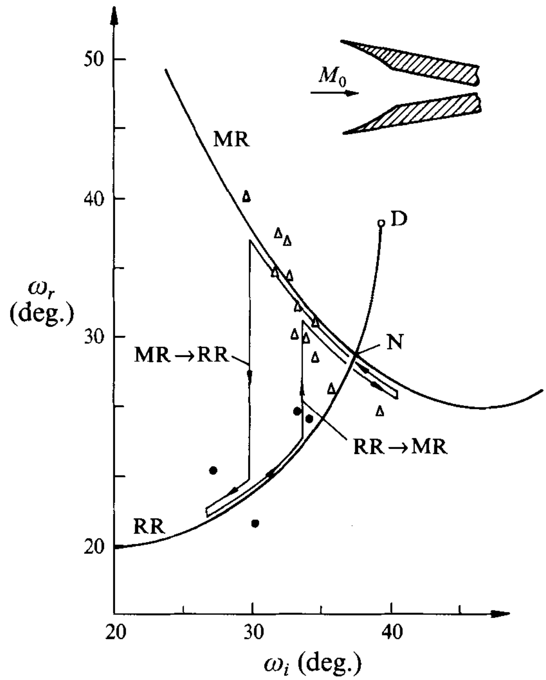

FIGURE 13. Henderson & Lozzi's (1979) experimental results over curved reflecting surface in the (ω_i, ω_r)-plane: △, MR configurations; ●, RR configurations.

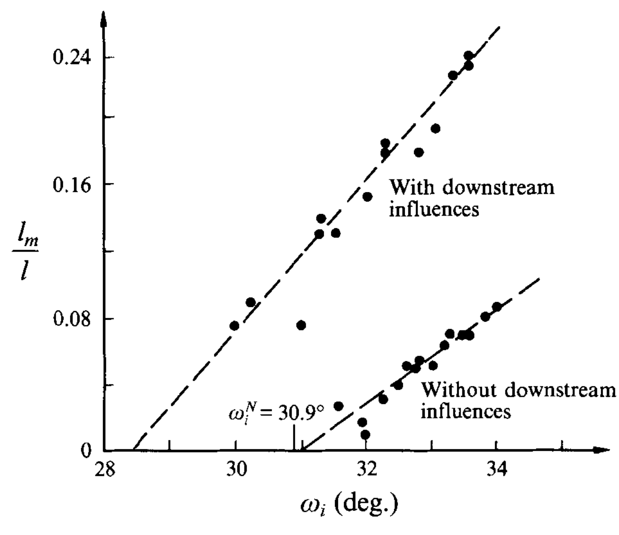

FIGURE 14. The dependence of the non-dimensionalized Mach stem height l_m/L on the incident shock wave angle ω_i for two cases with and without downstream influences.

imental facility and set-up, which was later used by Hornung & Robinson (1982), were such that the RR→MR transition occurred near or at the ω_i^N point. As a result, Hornung & Robinson (1982) mistakenly concluded that $\omega_i^{tr}(\mathrm{RR}\rightarrow\mathrm{MR}) = \omega_i^N$. Based on the present experimental results the transition criteria in steady flows are $\omega_i^{tr}(\mathrm{MR}\rightarrow\mathrm{RR}) = \omega_i^N$ and $\omega_i^N \leqslant \omega_i^{tr}(\mathrm{RR}\rightarrow\mathrm{MR}) \leqslant \omega_i^D$.

It should be noted here that Azevedo (1989), who obtained Hornung & Robinson's

(1982) raw experimental data, showed that the presence of downstream influences at the triple point caused an increase in the height of the Mach stem and, as a result, a shift in the MR → RR transition to values of $\omega_i^{tr}(\mathrm{MR} \rightarrow \mathrm{RR}) < \omega_i^N$. This is shown in figure 14 in which Hornung & Robinson's unpublished experimental results, as taken from Azevedo's (1989) PhD thesis, for cases with and without downstream influences, are shown. Note that while the results, which were free of downstream influences, indicated that $\omega_i^{tr}(\mathrm{MR} \rightarrow \mathrm{RR}) = 31° \approx \omega_i^N$, the results for which downstream influences were present indicated that $\omega_i^{tr}(\mathrm{MR} \rightarrow \mathrm{RR}) \approx 28.5° < \omega_i^N = 30.9°$. Consequently, it could be concluded that the MR → RR transition occurs at the von Neumann point, i.e. $\omega_i^{tr}(\mathrm{MR} \rightarrow \mathrm{RR}) = \omega_i^N$ provided the reflection is free of downstream influences. The fact that Hornung & Robinson (1982) obtained MR wave configurations for values of $\omega_i < \omega_i^N$ where, based on the three-shock theory for inviscid flows, MR wave configurations are impossible, is yet to be explained.

The authors would like to thank Dr Jean Claude Lengrand, Head of Laboratoire d'Aerothermique du CNRS, Meudon, France, for his encouragement and constructive remarks throughout the course of this study.

REFERENCES

ANDERSON, J. D. 1982 *Modern Compressible Flow*. McGraw-Hill.

AZEVEDO, D. J. 1989 Analytical prediction of shock patterns in a high-speed wedge bounded duct. PhD thesis, Dept. Mech. & Aero. Engng, State University of NY Buffalo.

AZEVEDO, D. J. & LIU, S. L. 1993 *AIAA J.* **31**, 83–90.

BEN-DOR, G. 1987 *J. Fluid Mech.* **181**, 467–484.

BEN-DOR, G. 1991 *Shock Wave Reflection Phenomena*. Springer.

BEN-DOR, G., TAKAYAMA, K. & KAWAUCHI, T. 1980 *J. Fluid Mech.* **100**, 147–160.

COURANT, R. & FRIEDRICHS, K. O. 1948 *Supersonic Flow of Shock Waves*. Wiley-Interscience.

HORNUNG, H. G. & KYCHAKOFF, G. 1977 In *Proc. 11th Intl Symp. Shock Tubes & Waves, Seattle, Washington, USA*, pp. 296–302.

HENDERSON, L. F. & LOZZI, A. 1975 *J. Fluid Mech.* **68**, 139–155.

HENDERSON, L. F. & LOZZI, A. 1979 *J. Fluid Mech.* **94**, 541–559.

HORNUNG, H. G., OERTEL, H. & SANDEMAN, R. J. 1979 *J. Fluid Mech.* **90**, 541–560.

HORNUNG, H. G. & ROBINSON, M. L. 1982 *J. Fluid Mech.* **123**, 155–164.

LANDAU, L. D. & LIFSHITZ, E. M. 1987 *Fluid Mechanics*. Pergamon.

LI, H. 1995 Reconsideration and modification of analytical models of shock and detonation wave reflections. PhD thesis, Dept Mech. Engng, Ben-Gurion University of the Negev.

LIEPMANN, H. W. & ROSHKO, A. 1957 *Elements of Gasdynamics*, John Wiley and Sons.

NEUMANN, J. VON 1963 *Collected Works* (ed. A. H. Taub), Vol. 6. Pergamon.